When Revolution Conflicts with Resolution: A Contemporary Issue Based on the Revisit of a Modern Compositional Approach

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INTRODUCTION

Responding to a common feature in atonal music—the lack of consistent sound organization—Charles Seeger pioneered a pre-compositional method called dissonant counterpoint (1913). In Seeger's method, all consonances must be prepared and resolved by dissonances, and to distinguish the quality of each harmonic interval, Seeger categorized all intervals into consonances or dissonances. However, like many other distinguished composers and music theorists (such as Ernst Krenek, Olivier Messiaen, and Joseph Straus), Seeger, somewhat problematically, questions the quality of the tritone and intentionally leaves this particular interval undefined. If the tritone is defined as neither a consonance nor a dissonance, its use in a chord will affect the quality of the corresponding harmony, making it indeterminate as well.

This begs the questions: If Seeger's dissonant counterpoint carries such an inherent problem, how do composers put this pre-compositional theory into practice? Would their derived compositions project a similar image, preserving this problem and presenting a succession of chords with ambiguous harmonic qualities? Or, is there an alternative solution that can be further adopted to solve this problem of the inherent quality of the tritone? To more clearly describe the problem this article proposes to analyze and compare two passages (mm. 20–29 and 49–56) from the third movement of Ruth Crawford's dissonant contrapuntal composition, the *String Quartet* from 1931.

In 1929 Crawford started her compositional lessons with Seeger, who would become her husband three years later. Her music—such as *Four Diaphonic Suites* (1930) and *Suite for Wind Quintet* (1952)—exhibits a deep influence from her instructor's method. The two passages from the *String Quartet* for my analysis were composed after two years of study with Seeger. On the surface level, these two passages share several common contextual features, implying a similar musical progression. However, my harmonic analyses, which are based on Seeger's pre-compositional rules and classifications of intervals, challenge this similarity by showing various and contrasting results that highlight the ambiguity of the passages, ambiguity arising from the uncertain quality of the tritone. The first analysis considers the tritone as a dissonance and derives two identical harmonic progressions, suggesting these two passages to be exactly alike. They both begin and end with an extremely dissonant harmony, reserving the most relatively consonant ones in the middle. The second analysis reconsiders the quality of the tritone as a consonance and posits two completely different harmonic progressions, suggesting these two passages to be dissimilar. This lack of a firm definition of the tritone quality provokes two conflicting analyses, and without a fixed assignment of the tritone quality, determining which analysis is correct or more analytically attractive becomes impossible.

Confronted with this problem, the second half of this article proposes a new harmonic measurement that can consistently measure chords. I call it the "average transposed interval class set" (*average-TVPicset*). Since neither consonance nor dissonance can describe the quality of the tritone, the *average-TVPicset* takes into account an alternative perspective of intervals—their spaces—and creates a simple formula that calculates the *degree of compactness* of a chord. The derived result is a decimal number range from 0.00 to 6.00. Within this range, the smaller the number, the more compact the harmony; the larger the number is, the more spatial the harmony. With the aid of harmonic measurement, no chord is ambiguous because it can always be defined as spatial or compact based on its corresponding decimal number. Finally, I apply the *average-TVPicset* to re-analyze the same two passages from Crawford's *String Quartet*. While the passages share several common contextual features, implying two similar musical progressions, my analysis explores their underlying harmonic structures, and thus reveals a deeper similarity, or perhaps a more significant difference, between them.

CHARLES SEEGER'S DISSONANT COUNTERPOINT

At the turn of the twentieth century, many composers attempted to progress beyond the confines of traditional tonality. To put such a new compositional project into practice, composers, such as Arnold Schoenberg, created a new style of composition, one in which dissonant and consonant intervals seem to share equal structural importance. We commonly refer to this type of composition as *atonal music*. From a creative perspective, a number of composers argued that they emancipated dissonances, whose interpretation no longer depended on the role of consonances; their works thus featured both dissonant and consonant structural intervals. From a critical perspective, their practice was not accompanied by a consistent arrangement of sound organization. The lack of a rigorous or consistent sound organization was judged as the greatest weakness in atonal music that required a highly rational system that could consistently organize this new type of composition.

In line with this criticism, Charles Seeger offers a profound critique of contemporary atonal composition. He judges it as "an elaboration and extension of the old diatonic and chromatic harmony rather than a revolutionary reversal of it."[1] He argues that for the past one thousand years, a steadily increasing number of consonant intervals have been accepted.[2] Consequently, intervals that are considered dissonant today may be reconsidered consonant in the future. Thus, the distinction between consonance and dissonance can be viewed as subjective or relative, and this type of problematic composition—"an elaboration and extension of the old diatonic and chromatic harmony"—could at some point be regarded as "consonant writing."[3]

To carry on the early twentieth-century compositional project with a more "revolutionary" and consistent approach that could reverse the former diatonic and chromatic harmonies, Seeger, in 1913, proposed a systematic contemporary method that he termed *dissonant counterpoint*. Dissonant counterpoint is a pre-compositional design containing rules that a composer must follow to create a musical work. In Seeger's system, contrary to tonal and modal counterpoint, it is the consonances that need to be properly prepared and resolved. A composition that is organized according to dissonant counterpoint will always feature one type of structural chord mainly composed of dissonant intervals. Extending this structural chord forms a harmonic prolongation that also characterizes the same chordal quality—dissonance. The hierarchy of intervals in which dissonances are superior to consonances is central to Seeger's method, because the distinction between the categories of consonance and dissonance is, at the same time, reinforced. Grounded in this hierarchical framework and the distinction between consonance and dissonance, a composition that is based on Seeger's method can never be considered "consonant writing."

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1. Seeger 1930, *Modern Music* 7: 25. In this article, Seeger mentions specifically compositions by Arnold Schoenberg and Alexander Skriabin.

2. Initially, according to Seeger, the only consonant intervals were perfect ones: the fourth, fifth and octave. By the end of the fourteenth century, the category of consonances included thirds and the major sixth. Finally, the dominant seventh joins the consonant family in the mid-nineteenth century. For the detailed discussion, see Seeger (1930) "Dissonance and the devil: An interesting passage in a Bach Cantata," *The Baton* 9:7–8.

3. Seeger 1930, 25.

The distinction between the categories of consonance and dissonance requires a precise classification that clearly defines and identifies the quality of each harmonic interval. To satisfy this requirement, Seeger classifies all intervals into consonances or dissonances. Before examining his classification of intervals, I will discuss the species and rules of dissonant counterpoint, and how to apply this particular pre-compositional method to a progression of harmonic intervals. Alongside Seeger's classification of intervals, I will evaluate this pre-compositional method, and identify the prominent features and inherent problems therein.

Within two-voice counterpoint, Seeger categorizes three species of dissonant counterpoint. Each species represents the number of notes in one voice that are related in time to a longer note in another voice. The first species illustrates two short notes that are against one long note; he terms this "two-way" counterpoint. The second and third species are, respectively, three notes against one, and four (or more) notes against one, termed "three-way" and "four-way" counterpoint. There are thirteen rules that composers must follow to practice Seeger's three species counterpoint. He calls these rules the *general procedure* (of dissonant counterpoint) that addresses two major fields: the principles of the intervallic progression and the overall musical texture.[4] Since my article focuses on the issue of the vertical dimension of a composition (harmony), I will discuss specifically the principles of intervallic progression in relation to the three species of dissonant counterpoint that include five rules, shown in Example 1.

4. Seeger 1994, 203–4.

(3) Preparation and resolution of consonance means *dissonating the consonance*. Melodically this is best done by skip.

(4) In the First Species the composer should alternate consonance and dissonance.

(5) In the Second Species, any two of three relationships…may be consonant provided they are well dissonated.

(6) In the Third Species, four-way relationship may contain three consonances and five-way may contain four. Only for special effect should there be more than four consecutive consonances, and then they must be very well dissonated…

(8) The restrictions of paragraph (4), (5), and (6) can sometimes be slightly modified…

Example 1. Seeger 1994, 203. General Procedure of Dissonant Counterpoint; rules 3, 4, 5, 6, and 8.

According to rule number 3, unlike tonal and modal counterpoint, the pitch that prepares and resolves the harmonic consonance, moves by leap instead of by step. Rules 4–6 are particularly important because they provide clear guidelines for composers to add intervals one after another. While the last two species can have two to four consecutive consonances (rules 5 and 6), the first species must strictly alternate the consonant and dissonant intervals. To follow the above guidelines for consecutive intervals, composers must be well informed or acutely aware of the distinction between consonances and dissonances. Therefore, Seeger also provides his personal classification of all harmonic intervals, shown in Example 2.

Perfect	1 (*ic* 0)	Perfect Consonances
Perfect	8 (*ic* 0)	
Perfect	5 (*ic* 5)	
Perfect	4 (*ic* 5)	
Major	3 (*ic* 4)	Imperfect Consonances
Major	6 (*ic* 4)	
Minor	3 (*ic* 3)	
Minor	6 (*ic* 3)	
Tritone	(*ic* 6)	More consonant chordally than melodically
Major	2 (*ic* 2)	Imperfect Dissonances
Minor	7 (*ic* 2)	
Major	9 (*ic* 2)	
Minor	2 (*ic* 1)	Perfect Dissonances
Major	7 (*ic* 1)	
Minor	9 (*ic* 1)	

Example 2. Seeger 1994, 30. Harmonic Consonances and Dissonances; each interval is translated into an interval class indicated in parentheses.

According to this example, there is the primary division between consonances and dissonances, and a secondary division within each category. This secondary division classifies consonant and dissonant intervals into perfect and imperfect groups.[5] Translating all the intervals into interval classes, each group in the consonant category has two members: *ics* [0, 5] that appear in the group of perfect consonances while those of [3, 4] are in the imperfect. The remaining three *ics* of 1, 2 and 6 can be found, respectively, in the groups of perfect and imperfect dissonances, and the tritone. Apparently, there are more *ics* in the consonant category (*ics* 0, 3, 4, 5) than those in the dissonant (*ics* 1 and 2). Although only two *ics* appear in the

5. In the following discussion, the term "quality" means the degree of consonance and dissonance of an interval, while "condition" refers to the degree of perfection and imperfection of an interval.

dissonant category, each by itself represents a special group of dissonance (*ic* 1 for the perfect dissonance, *ic* 2 for the imperfect dissonance). This secondary division of the dissonant category thus reveals Seeger's particular interest in dissonances.

While Example 2 seemingly defines the quality of all harmonic intervals, it leaves one particular interval undefined—the tritone. It is the only interval not included in either interval category. Instead of assigning a particular quality, Seeger provides a suggestive comment on the tritone stating that it sounds "more consonant chordally than melodically."[6] Seeger's suggestive comment leaves one pondering the following questions: Is the tritone a consonant interval? If so, to which condition (perfect or imperfect) does it belong? Or, more consistent with traditional theories, is Seeger's conception of the tritone still a dissonance? If so, why does he not clarify its quality? The issues become more puzzling when contemplating the practical use of Seeger's classification of intervals: How do composers apply Example 2, in which the quality of the tritone lacks a fixed assignment, to practice the three species of dissonant counterpoint? Do they consider the tritone a dissonance that prepares and resolves consonant intervals? Or do they consider it a consonance that is prepared and resolved by a dissonant interval? To envision these questions concretely, I use Ruth Crawford's dissonant contrapuntal composition, *String Quartet*, as a case study, and analyze two passages from the third movement. This analysis discovers how Crawford puts this somewhat problematic pre-compositional theory into practice. In return, perhaps there can be found a possible and proper justification that can be adopted to explain or describe the quality of the tritone in Seeger's dissonant counterpoint.

6. Seeger 1994, 130.

ANALYSIS OF RUTH CRAWFORD'S *STRING QUARTET*

Example 3. Crawford 1931. *String Quartet*, Movement III, measures 20–29. Segmentations of the six harmonic tetrachords.

In the third movement of Crawford's *String Quartet*, each instrument, for the most part, plays a single and different pitch. A succession of four different simultaneous pitches creates a harmonic progression that unfolds a series of tetrachords. I have selected a passage of ten measures from measure 20 (where the first full tetrachord enters) to measure 29, which appears along with the harmonic segmentation in Example 3. There are six tetrachords in this example, in which each pair of adjacent chords has three pitches in common; the uncommon pitches appear in either violin I or in the cello parts. This passage begins and ends with an *sc* 4-1, and creates an implied tetrachordal prolongation. Before exploring this harmonic prolongation, I will demonstrate "two-way" species dissonant counterpoint between the two outer and comparatively more active voices (*Vln I* and *Vc*) in Example 3.[7]

In order to create a clearer image of the two outer voices, Example 4 removes the two inner instruments (*Vln II* and *Vla*) from Example 3, and presents only *Vln I* and *Vc*. Comparing the melodic lines of these two voices, the first violin plays a melodic major third, in which $F^{\#}_{4}$ (mm. 21–25) intervenes between the two boundary pitches, D_4 (mm. 20–21 and mm. 25–29) and creates a symmetrical up-down motion. This motion is imitated by the cello, but with some slight differences. Instead of a major third, the central pitch E_4 (mm. 22–24) is only a minor third from the first and final $C^{\#}_{4}$ (mm. 20–22 and mm. 26–29). In addition, B_3 (mm. 24–26) mediates between E_4 and the last $C^{\#}_{4}$, and deviates from the direct descent of the minor third. Thus, E_4 first

7. Throughout Example 3, the inner two voices form a single dyad (E^{b}_{4} in *Vln II* and C_4 in *Vla*) that accompanies the remaining two outer and more active voices. According to the number of pitches, I divide the four string voices into two groups: the static (*Vln II* and *Vla*, each has one single pitch) and the active (three pitches in *Vln I* and four in *Vc*). Combining one voice from the first group with another from the second derives four additional pairs of voices, shown in Table F-1. The top row and the left column represent the voices of the first and the second groups, respectively. Reading from left to right, top to bottom, the four new pairs of voices are further categorized into Boxes A [(*Vln I*, *Vln II*) and (*Vln I*, *Vla*)] and B [(*Vc*, *Vln II*) and (*Vc*, *Vla*)]. While each pair of voices in Box A produces a single unit of the second species (three-way: three short notes in *Vln I* against a long note in *Vln II* or *Vla*), combining the cello with the second violin or viola results in another single unit but with a different species—the third (four-way: four short notes in *Vc* against a long note in *Vln II* or *Vla*).

One problem arises if we put the two voices next to each other as seen in Table F-1: each pair only produces *one unit* in relation to its corresponding species. This problem hinders the understanding of how Crawford in fact accurately applies Seeger's species of dissonant counterpoint to combine two simultaneously *autonomous voices*. Consequently, I turn my focus to the voices in the second group (the outer voices of this string quartet), since they are the only two moving voices in this passage.

Group I / Group II	***Vln II***	***Vla***	
Vln I	(*Vln I*, *Vln II*)	(*Vln I*, *Vla*)	**Box A**
Vc	(*Vc*, *Vln II*)	(*Vc*, *Vla*)	**Box B**

Table F-1. Combination of the two contrapuntal voices in Example 3; each pair of voices in Box A produces a single unit of Seeger's three-way counterpoint, that in Box B produces a single unit of four-way counterpoint.

descends five semitones to B_3, and then continuously moves up a whole-step to $C^{\#}_4$. Next, I use boxes to mark the *consecutive pitches* that have *comparatively long rhythms*.

Example 4. Crawford 1931. *String Quartet*, Movement III, measures 20–29; *Vln I*, *Vc.*

Example 5. Crawford 1931. *String Quartet*, Movement III, measures 20–29; *Vln I*, *Vc*.
Boxes mark the pitches with comparatively long rhythms.

The results, in Example 5, show the marked pitches $C^{\#}_4$ in *Vc* (mm. 20–22), $F^{\#}_4$ in *Vln I* (mm. 20–25), and D_4 in *Vln I* (mm. 25–29).[8] Observing the remaining unmarked pitches whose rhythms are comparatively short, we find that there are consistently two unmarked pitches

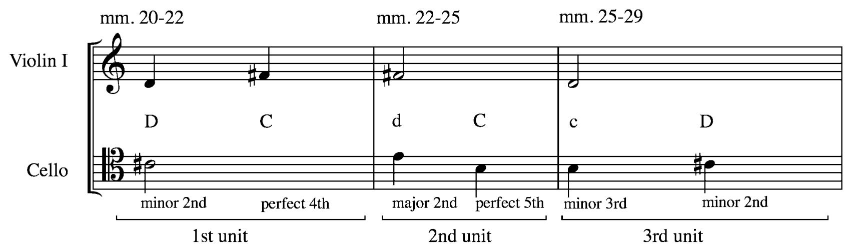
8. Here, and in the following discussion, the term "consecutive pitches" refers to any two adjacent but not overlapping pitches. In other words, pitches must progress one after another and produce a flowing movement. Thus, although $F^{\#}_4$ in *Vln I* appears on the fourth beat of m. 21, to avoid overlapping this pitch with that in the first box of Example 5 ($C^{\#}_4$, *Vc*), I do not box it until the cello ceases playing the $C^{\#}_4$. The second box that marks $F^{\#}_4$ in *Vln I* starts on the third beat of m. 22.

against a marked one. Thus, in terms of their associated long and short rhythms, there are consistently two short pitches against a long one. Based on the above description, these pitches correspond to one of Seeger's three species of dissonant counterpoints: the two-way counterpoint (two short notes against a long note). The discovery of this two-way counterpoint between the outer two voices from Example 3 leads to another crucial question: Does Crawford strictly follow rule number 4 in Seeger's general procedure (Example 1), which requires alternation of consonant and dissonant intervals in this type of species counterpoint?

In response to this question, Example 2 must be applied to examine the consonant or dissonant intervals formed by these two voices. To recreate a simplified image of two-way counterpoint between *Vln I* and *Vc*, Example 6 shows half notes and quarter notes to represent, respectively, the long (marked) and short (unmarked) notes in Example 5, and thus derives three units of this specific species counterpoint. Letters between the staves indicate the quality (consonant: *C*/*c*, dissonant: *D*/*d*) of the six intervals; upper and lower letter cases indentify their condition (perfect: *C*/*D*; imperfect: *c*/*d*). The rules mentioned in the following discussion refer to those listed in Example 1. According to rule number 4, Crawford strictly alternates the consonance and dissonance within each unit. The first two units end with the letter *C*—the perfect consonances of a perfect fourth and perfect fifth. Each of these two consonances is well prepared by a dissonance through the leap in one voice.[9] While the perfect fourth in the first unit is resolved by a major second (the cello ascending a minor third to E_4), the perfect fifth in the second unit is not resolved until the end of the passage. Along with D_4 in the first violin (in the third unit), the repetition of B_3 in the cello results in another imperfect consonant interval, a minor third. By ascending B_3 a whole-tone up to $C^{\#}_4$, this new imperfect consonance is finally

9. In the first unit, D_4 in the first violin ascends a major third to $F^{\#}_4$, and in the second unit, E_4 in the cello moves five semitones down to B_3.

resolved by a perfect dissonance, a minor second. Thus, except for the third unit, Crawford generally complies with Seeger's rule number 3, introducing and resolving a consonant interval by the leap in one voice.



Example 6. Half and quartet notes represent long and short notes from Example 5.

From this analysis, we gain a more concrete sense of Seeger's species dissonant counterpoint as it structures free composition. When the violin II and viola are included in the analysis, the four simultaneous contrapuntal voices create a harmony. According to Example 3, this musical passage is structured by six tetrachords; successively: *scs* 4-1, 4-13, 4-12, 4-18, 4-3, and 4-1. As pointed out earlier, this musical passage begins and ends with an *sc* 4-1, and can thus be judged as a prolongation of the particular tetrachordal set class of 4-1. Although this judgment—which relates two non-adjacent chords based on their identical *sc* names to form a harmonic prolongation—is not uncommon in a post-tonal analysis, it nevertheless overlooks important details within Example 3.[10] I pose the following questions to describe these details: How do *scs* 4-13, 4-12, 4-18, and 4-3 realize this prolongation? What are the features and

10. In music set theory, several unordered *pcs* form a pitch class set (*pcset*) that is identified by particular interval class content. This content has six numbers, in which the first gives the number of occurrences of *ic* 1, the second gives that of *ic* 2, and so forth. This content is generally referred to as the *interval class vector*. With the transposition and inversion of a *pcset*, we can derive more *pcsets* of different *pcs*, which supposedly enjoy a similar sound quality in accordance with their identical interval class vector. Following Allen Forte's method, all these resultant *pcsets* can be further categorized into the same form, called a *set class* (e.g., *sc* 4-1). As a result, two chords with the same *sc* name will always share an identical interval class vector and, thus create a similar sound quality. Based on this similar sound quality, we commonly relate non-adjacent chords with the same *sc* name as forming a particular harmonic prolongation.

processes that appear on the musical surface before the music achieves its prolongational goal? With regard to these questions, one must find a way to reinterpret all the tetrachords in Example 3 by observing the relationships among them. Since this article concentrates on the perspective of the quality of an interval (or interval class), I suggest an extension of this perspective in order to measure the consonance or dissonance of a harmony. By "harmony" I refer to a group of *pcs* that are consistently *ordered according to their different timbres, from bass to treble instrumental voices*.[11] Because this harmonic perception relies fully on the different instrumental timbres, the relationship between the immediately adjacent voices becomes far more intimate and direct than that between the non-adjacent ones. Thus, later in my discussion, I will take into account only the adjacent instruments and exclude the non-adjacent ones. The distance, or an *ic*, between a pair of adjacent voices is called "voice-pair interval-class" (*VPic*); several *VPics* form a set, termed "voice-pair interval-class set" (*VPicset*). Since there are four simultaneous voices moving throughout Example 3, the *VPicset* that represents a harmony in this example will always include three *VPics*, each of which, from low to high, appears between the adjacent voices of [*Vc–Vla*], [*Vla–Vln II*] and [*Vln II–Vln I*].

In traditional counterpoint, a harmony that possesses the most dissonant intervals is considered a dissonant harmony, producing instability and tension. In contrast, the opposing sonic quality—stable and relaxed—is created by a consonant harmony that includes the least dissonant intervals. By extending this traditional concept of consonant and dissonant harmonies, I determine that the quality of a chord in Example 3 is based primarily on said concept, using the proportion of the consonant *VPics* to the dissonant ones within a *VPicset* to define consonant and dissonant harmonies. For instance, while one chord, whose *VPicset* comprises a relatively large

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11. For example, following Robert Morris' notation, the ordered *pcset* of the second tetrachord, *sc* 4-13, in Example 3 will be <1, 0, 3, 6>, where the four *pcs* are successively arranged according to their different timbres from the lowest to the highest string instruments, <*Vc*, *Vla*, *Vln II*, *Vln I*>.

number of consonant *VPics* forms a consonant harmony, another chord, whose *VPicset* comprises a relatively large number of dissonant *VPics* forms a dissonant harmony. Grounded in this definition of harmonic quality, I measure the qualities of the six tetrachords in Example 3 by comparing their numbers of consonant and dissonant *VPics*.

Example 7 presents the six tetrachords from Example 3; their corresponding *VPicsets* appear directly below the musical system. Within each *VPicset*, the numbers, reading from bottom to top (or left to right in the text), indicate the *VPics* in one chord.[12] Similar to my discussion of Example 6, letters appearing to the right of the *VPics* represent the qualities and conditions of their associated *ics* in Seeger's classification of intervals.[13] Throughout Example 7 there are only two types of intervals that form these six tetrachords: imperfect consonances (*c*: *ics* 3 and 4) and perfect dissonance (*D*: *ic* 1). The bracketed number located below the *VPicset* shows how many perfect dissonances are found in each chord (see line A). Three chords have the greatest number (2) of perfect dissonance: the first (*sc* 4-1) and the last two (*scs* 4-3 and 4-1). Set class 4-12 in the middle of the progression comprises three imperfect consonances, that is, a set without dissonances. The remaining two chords (the second and the fourth) have one perfect dissonance and two imperfect consonances. Each chord is assigned with a particular quality based on its corresponding bracketed number (see Table 1). The largest number makes a dissonant harmony; the smallest makes a consonant one. The number in between represents a medial harmony.

12. The *VPics* for the second tetrachord, *sc* 4-13, in Example 7 are 1, 3, and 3 (from bottom to top). These three *VPics* form a *VPicset* of [1, 3, 3].

13. In my discussion, a *VPic* specifically refers to an interval class between a pair of adjacent voices, while an *ic*, generally refers to a category of an interval class.

Example 7. Crawford 1931. *String Quartet*, Movement III, measures 20–29; *VPicsets*. Line A shows the amount of dissonant *VPics* in each *VPicset.*

Bracketed Numbers in Example 7	[0]	[1]	[2]
Corresponding Harmonic Qualities	**Consonant**	**Medial**	**Dissonant**

Table 1. Qualities of the tetrachords in Example 7 are defined based on their corresponding bracketed numbers.

Example 7 starts with a dissonant harmony (*sc* 4-1) and crosses a medial harmony (*sc* 4-13) to a consonant harmony (*sc* 4-12) halfway through the passage. Set class 4-12 moves to another medial chord (*sc* 4-18) before it arrives at the final two dissonant chords, *scs* 4-3 and 4-1. The entire progression forms a complete prolongation of a dissonant harmony (*sc* 4-1) with the greatest number of perfect dissonances elaborated by a consonant chord (*sc* 4-12) and two medial chords (*scs* 4-13 and 4-18).

Unlike Example 3 that shows only the large-scale harmonic prolongation of *sc* 4-1 (see my earlier discussion), the analysis of Example 7 reveals all the details of the harmonic progression in Crawford's passage, and demonstrates how she embellishes and prolongs the dissonant harmony of this particular set class. To make a further comparison with Example 7, a musical passage twenty measures later (mm. 49–56) with its harmonic segmentations is presented in Example 8.[14] Similar to the previous passage, Example 8 begins with *scs* 4-1 and 4-13 articulated in 4/4 meter (Crawford constantly alternates meters of 3/4, 4/4 and 5/4 in this movement). After the initial two *scs* 4-1 and 4-13, the music passes through two other harmonies, *scs* 4-11 and 4-4, before arriving at the final chord, *sc* 4-Z15. Comparing this final tetrachord with the one in Example 7, they do not share the same *sc* name (*sc* 4-1 in Example 7 and *sc* 4-Z15 in Example 8), yet they both span a long duration of four measures and create a stable motion, or a sense of closure. Based on the above observation of the common contextual features between Examples 7 and 8 (meter of 4/4, long duration of the final chord, and the initial *scs* 4-1 and 4-13) these two successive examples can be regarded, at this moment, as unfolding two similar musical progressions.

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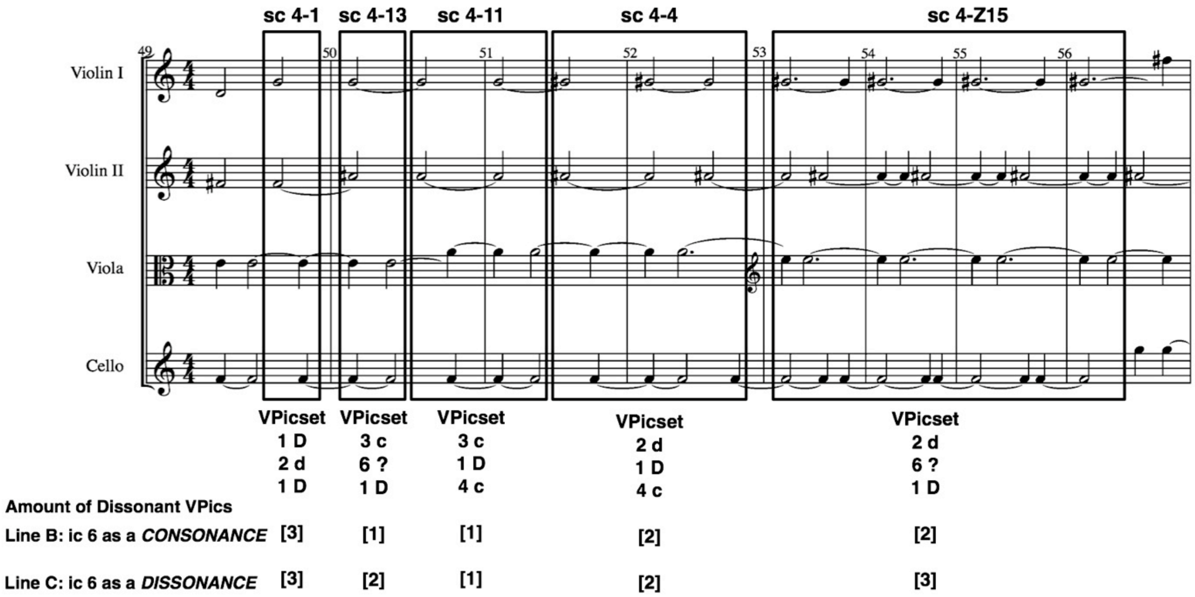
14. Like the tetrachords in Example 7, each pair of the adjacent harmonies in Example 8 share three common tones; uncommon tones appear in *Vln I*, *Vln II*, or *Vc*.



Example 8. Crawford 1931. *String Quartet*, Movement III, measures 49–56.
Segmentations of five tetrachords, corresponding *VPicsets*, and amount of dissonant *VPics* (lines B and C).

Bracketed Numbers in Ex. 8	[1]	[2]	[3]
Corresponding Harmonic Qualities	**Consonant**	**Medial**	**Dissonant**

Table 2. Qualities of the tetrachords in Example 8 defined based on their corresponding bracketed numbers.

Aside from these contextual features, is there a deeper and more grounded common element—such as harmonic progression—that relates Examples 7 and 8? Do these two passages project an identical harmonic progression? Or, on the contrary, do they project two different harmonic progressions? To answer these questions, I analyze the harmonies in Example 8 with the same approach for those in Example 7, and then compare the resultant harmonic progressions between these two examples. Below each tetrachord in Example 8 appear two items: the

VPicsets and the bracketed numbers. The bracketed numbers (lines B and C) show the amount of dissonant *VPics* within their corresponding *VPicsets*; the letters *C*, c, *D*, *d* to the right of their corresponding numbers represent the harmonic qualities and conditions.[15] Observing the five *VPicsets* in Example 8, one notices the two special chords of *scs* 4-13 and 4-Z15 that contain a tritone (the interval without clear harmonic quality and condition). Within these two chords, their corresponding tritones appear between the two inner voices of the viola and violin II. Thus, before analyzing the complete harmonic progression of Example 8, my first step is to observe Crawford's personal view on this interval and to examine how she uses it to compose these two contrapuntal voices.

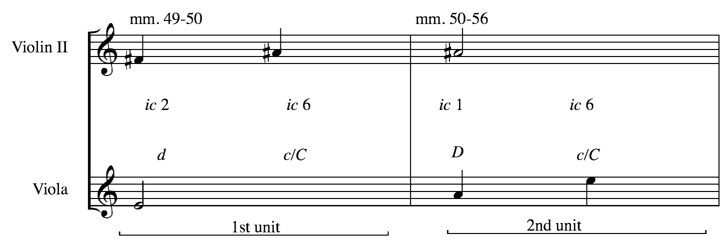
Example 9 removes the outer voices of *Vc* and *Vln I* from Example 8. The boxes mark the consecutive pitches with comparatively long durational values, while the unmarked ones have comparatively short durations. Half notes and quarter notes represent, respectively, the marked (long value) and unmarked pitches (short value) that derive two units of the two-way species counterpoint, shown in Example 10. There is one dissonance in each of the two units: the imperfect in the first unit (*ic* 2) and the perfect in the second unit (*ic* 1). In this particular species, according to Seeger's rule number 4, consonance and dissonance must alternate. Since each unit already contains one dissonance, the tritone here must be a consonance that alternates between *ics* 2 and 1.[16] This brief analysis shows Crawford's personal and innovative view of *ic* 6, which she assumes as a representative of a consonance. At present, I will follow Crawford's view of the

15. Since there lacks fixed assignment of the tritone's harmonic quality, a question mark placed to the right of *ic* 6 highlights the ambiguous quality and condition of this interval.

16. Presumably, *ic* 6 is a consonance, though its condition (perfect or imperfect) remains uncertain. Therefore, instead of assigning a particular condition to the tritone, both letter cases *C* (perfect consonance) and *c* (imperfect consonance) are indicated in Example 10.

tritone and place this interval to the consonant category.[17] I then apply this new classification of intervals to examine the consonance or dissonance of the five harmonies in Example 8.

Example 9. Ruth Crawford, *String Quartet*, Movement III, measures 49–56; *Vln II*, *Vla* (1931). Boxes mark the consecutive pitches with comparatively long durational values.



Example 10. Half and quarter notes represent the long and short notes of Example 9.

Below each *VPicset* in Example 8 there are two lines that contain bracketed numbers that provide the amount of dissonant *VPics* within one chord. Line B follows Crawford's apparent view and considers the tritone a consonance; the dissonant *VPics* found in each tetrachord are 3, 1, 1, 2, and 2, respectively. Translating all five tetrachords into consonant, medial, and dissonant chords with respect to one, two, and three dissonant *VPics* (see Table 2), the result shows that the harmonic progression begins with a dissonant chord of *sc* 4-1 (three dissonant *VPics*) that suddenly moves to two consonant chords of *scs* 4-13 and 4-11 (one dissonant *VPic*) without a medial chord in between. In fact, there are two medial chords in Example 8, but they do not

17. Thus, the consonances here now include *ics* 0, 3, 4, 5, and 6, while the dissonances include *ics* 1 and 2.

appear until the end of this example; they are *scs* 4-4 and 4-Z15. Comparing the resultant harmonic progression in Example 8 to that in Example 7, two markedly different harmonic progressions are projected. While the bracketed numbers in Example 7 form a symmetrical pattern of 2–1–0–1–2–2 (line A) that represents a harmonic progression gradually moving away from a dissonant chord to a consonant chord and then returning to a dissonant chord, those in Example 8 form a seemingly irregular pattern of 3–1–1–2–2 (line B) where the dissonant and consonant harmonies appear in the beginning of the passage and push the medial harmonies to the end.[18] As a result, considering the tritone as a consonance derives two contrasting harmonic progressions between mm. 20–29 (Example 7) and mm. 49–56 (Example 8). Further, from a deeper and underlying harmonic perspective, the above comparison suggests that beneath the common contextual features, these two passages in Crawford's *String Quartet* are categorically dissimilar.

Before concluding my above analyses, I would like to pose two more questions: What does the harmonic progression look like if we stick to a more traditional view of the tritone and consider it a dissonance? Will this shift significantly affect the quality of a harmony and change our interpretation of the entire harmonic progression in Example 8? In response, I challenge Crawford's innovative view of the tritone and place this interval to the dissonant category.[19] Based on this new classification of intervals, I re-analyze Example 8 and derive a new line of five bracketed numbers on the bottom of this example (3–2–1–2–3, see line C). The result shows

18. In order to determine the consonance or dissonance of the harmonies, the bracketed numbers can only be compared with one another *within*—not between—their corresponding lines A and B (or lines A and C in the following discussion). For instance, the last bracketed number in line A (*sc* 4-1, see Example 7) is 2, which is identical to that in line B (*sc* 4-Z15, see Example 8). Although these numbers describe the same amount of dissonant *VPics*, they articulate—according to their corresponding musical contexts and my definitions of harmonic qualities in Tables 1 and 2—two different qualities for their associated chords. While the former represents a dissonant harmony in line A (*sc* 4-1, see Table 1), the latter becomes a medial harmony in line B (*sc* 4-Z15, see Table 2).

19. Thus, now the dissonant category includes *ics* 1, 2, and 6; the remaining ones, *ics* 0, 3, 4, and 5 appear as consonances.

that within line C, only two numbers differ from those corresponding in line B; they appear in the second and the last positions of these two lines. From lines B to C, the numbers in these two positions all increase one degree from one to two (the second position) and two to three (the last position). According to Table 2, using consonant, medial, and dissonant harmonies to describe the above-mentioned numbers, the harmony in the second position (*sc* 4-13) is transformed in quality from consonant to medial, and that in the final position (*sc* 4-Z15) is transformed from medial to dissonant. Unlike *scs* 4-13 and 4-Z15, the remaining bracketed numbers between lines B and C stay the same, and thus their associated tetrachords—*scs* 4-1, 4-11 and 4-3—do not change their corresponding harmonic qualities.

To summarize this discussion of Example 8, shifting the view of the tritone from a consonant to a dissonant interval alters only two bracketed numbers from line B to line C, thus transforming the harmonic qualities of their two associated chords—*scs* 4-13 and 4-Z15. Proportionally, two out of five chords of differing qualities between lines B and C may appear to have an insignificant influence on the entire harmonic progression in Example 8. However, if we apply these new assignments of harmonic qualities to *scs* 4-13 and 4-Z15 to re-interpret the harmonic progression in Example 8, the result contradicts the above assumption.

Based on Table 2 and translating the bracketed numbers in line C (3–2–1–2–3) into their corresponding consonant, medial, and dissonant harmonies, this passage now begins with a dissonant harmony (*sc* 4-1) and crosses a medial harmony (*sc* 4-13) to a consonant harmony (*sc* 4-11) halfway through the passage. Set class 4-11 moves to another medial chord (*sc* 4-4) before it arrives at the final dissonant chord, *sc* 4-Z15. The entire progression forms a dissonant harmonic prolongation of *scs* 4-1 and 4-Z15 elaborated by a consonant chord (*sc* 4-11) and two medial chords (*scs* 4-13 and 4-4). This harmonic progression is, in fact, familiar; it is almost an exact repetition as shown in Example 7, in which the bracketed numbers representing this example are 2–1–0–1–2–2 (see line A). The numbers in lines A and C form a symmetrical pattern and demonstrate a common harmonic progression that gradually alternates between dissonant and consonant harmony. Based on this symmetrical pattern, I argue that, aside from the common contextual features between Examples 7 and 8, the passages in these two examples also share a common harmonic progression. However, this argument conflicts with my earlier suggestion that posits these two passages as projecting two contrasting harmonic progressions (lines A and B). Therefore, though shifting the view of the tritone from a consonant to a dissonant interval causes some slight differences between lines B and C in Example 8, the result has a great influence on the entire harmonic progression, which is identical to the progression in Example 7.

Which analysis better represents the relationship between the two passages in Examples 7 and 8? Do we agree with Crawford's personal and innovative view of the tritone to recognize this interval as a consonance, and regard the two passages as projecting two contrasting harmonic progressions? Or, consistent with the traditional view of the tritone, do we recognize it as a dissonance and regard the two passages as projecting a common harmonic progression? It seems unavoidably clear that without a solid basis or fixed assignment for the quality of the tritone, these questions, crucial to perceiving the relationship between Crawford's two passages, cannot be answered. Is there a solution that can solve this impasse?

NEWLY DERIVED HARMONIC MEASUREMENT: *average-TVPicsets*

Every harmonic interval has two facets: its *quality*, which can be classified into a consonance or a dissonance, and its *space*, which can be translated into one of the seven interval classes from 0 to 6. In Seeger's dissonant counterpoint, the former cannot be applied when describing the tritone, and if neither consonance nor dissonance can explain the quality of the tritone, the consequence, according to my analysis of Crawford's *String Quartet*, seriously affects and obfuscates the perception of a harmonic progression that follows Seeger's method of dissonant counterpoint. To address this and form a better understanding of the harmonic progression in Crawford's *String Quartet*, I suggest taking into consideration the second facet of all intervals—their spaces (specifically, their interval classes)—and use them to create a simple arithmetic formula which can consistently measure *the degree of compactness* of chords in a dissonant contrapuntal composition.

To carry out this task, I return to Seeger's classification of intervals in Example 2 in which the consonant category contains *ics* 0, 3, 4, and 5, while *ics* 1 and 2 appear in the dissonant category. Generally, these six *ics*—consonances: *ics* {0, 3, 4, 5} and dissonances: *ics* {1, 2}—are ordered, as seen in Table 3, according to their numbers *from small to large*. In this table, the consonant *ic* 0 is separated from the other consonant members (*ics* 3, 4, and 5) by the two dissonant *ics* 1 and 2. If this consonant *ic* 0 is distant from the remaining consonant *ics*, then Table 3 obscures the clear boundary between consonances and dissonances in Seeger's classification of intervals. To avoid the obscurity and re-establish a clear boundary between consonances and dissonances in Table 3, a new ordering is devised that best mirrors and accommodates the framework of Example 2. Within this new ordering, all consonant and, likewise, dissonant *ics remain as close as possible*. Before presenting and demonstrating this new ordering of *ics*, I will explain the special property formed by the interval class group.

Interval classes range from 0 to 6 and numbers larger than 6 (7 to 12) are equivalent to their inversely related *ics* in mod 12. These seven sequential interval classes in Table 3 are recursive and form a closed group. In this closed group, if *ic* 0 is moved from the first position to the last (shown in Table 3), the sequential numbers begin with *ic* 1 and end with *ic* 0, and thus derive a new ordering of *ics* {1, 2, 3, 4, 5, 6, 0}, shown in Table 4; the solid arrow brings *ic* 0 from the first position to the last and the open arrow moves *ics* 1–6 to the left of the table. This new ordering of *ics* possesses two strengths that are lacking in Table 3. First, *ic* 0 is now only one element (*ic* 6) away from the other three consonant *ics* 3, 4, and 5, shown in Table 5. Grounded in the framework of Table 5, a bold, dashed vertical line and two pentagon-like figures are used to divide the seven *ics* into two primary groups: consonant and dissonant. The dissonances (*ics* 1 and 2) appear in the figure on the left of the table and the consonances appear in the larger figure on the right. Second, aside from the primary division between consonances and dissonances, Seeger's secondary division of intervals between the perfect and imperfect groups also remains clear.

ICs	0	1	2	3	4	5	(6)
small number → large number							

Table 3. Ordering *ics* from small to large numbers.

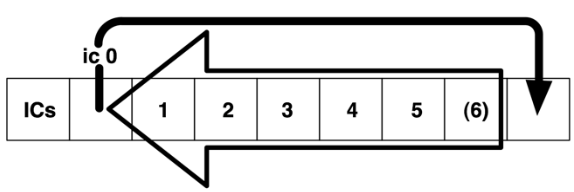


Table 4. The solid arrow moves *ic* 0 from the extreme left position to the extreme right in Table 3, and the open arrow pushes the remaining *ics* 1–6 to the left of the table.

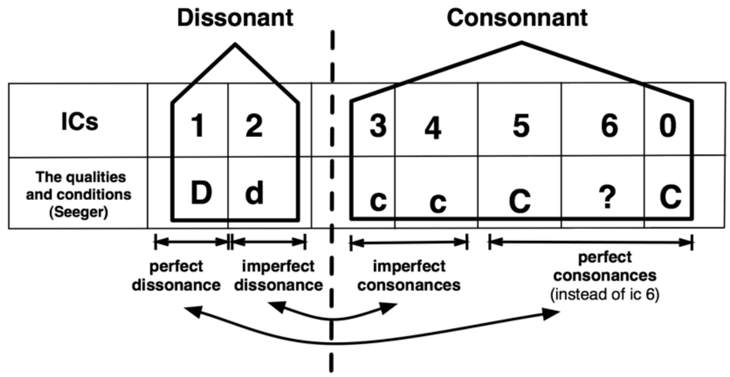


Table 5. Ordering *ics* {1, 2, 3, 4, 5, 6, and 0} based on Seeger's categorization of harmonic intervals in Example 2; arrows on the bottom show the symmetries between the perfect and imperfect groupings.

Following the bi-directional, bracketed arrows below letters *C*/*c*/*D*/*d* in Table 5, the imperfect consonances (*ics* 3 and 4, marked with *c*) are adjacent, while the perfect ones (*ics* 5 and 0, marked with *C*) have one element between them. The boundary between the perfect and imperfect consonances is now more apparent. Furthermore, the imperfect and perfect groupings (imperfect consonances/dissonances and perfect consonances/dissonances) form symmetries in this new ordering. Thus, Table 5 creates a clearer representation and mirrors the format of Seeger's classification of the harmonic intervals.

As stated earlier, interval classes are generally ordered from small to large numbers (0–6, as in Table 3). The numbers from 0 to 6 correspond to the space from the smallest to the largest, or to the *ics* from *the most compact* to *the most spatial*. In comparison with Table 3, the ordering of *ics* from 1 to 0 in Table 5 seems to be inconsistent with regard to their spaces. The largest and the smallest spaces (*ics* 6 and 0, respectively) are now adjacent to each other at the extreme right of Table 5, while the remaining *ics* (1–5) appear to the left. If, and only if, *ic* 0 is understood as *an octave* instead of a unison, this interval class becomes a larger space than that of the *ic* 6.[20] Table 6 represents the space of the seven *ics* in Table 5 in which the element on the extreme right will be the most spatial interval class (*ic* 0). At the other extreme, interval class 1 becomes the most compact *ic*. In this case, Table 6—an ordering of *ics* that is based on Seeger's classification of the harmonic intervals—simultaneously unfolds the "degree of compactness" of the *ics*. The numbers from 1 to 0 consistently correspond to the *ics* from the most compact to the most spatial, or to the space from the smallest to the largest.[21]

ics (VPics)	1	2	3	4	5	6	0
the degree of compactness	**Compact** ◄———						———► **Spatial**

Table 6. Degrees of compactness among *ics* {1, 2, 3, 4, 5, 6, 0}.

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20. For instance, in set theory, a clock is used to represent *pc-space* as shown in Figure F-1. The points, numbers 0-11 are the twelve *pcs* 0-11 (see the inner solid circle). The distance between any two *pcs* around the clock creates an interval (or interval class). To measure this distance, two strict rules are set up that must be followed in order to map one *pc* to another. First and foremost, no *pc* is allowed to remain stationary, and second, only the *shortest distance* (or the least steps) between two *pcs* is taken into consideration.

If a is mapped to itself (in Figure F-1), the shortest distance will be a full-circle progression that corresponds to an octave or *ic* 0, and creates the largest interval space in Figure F-1. For example, mapping *pc* 0 to another *pc* 0 by following the above rules, the first *pc* 0 must progress around the clock in order to return to its original point: the second *pc* 0 (see the dashed arrow moving in a counter-clockwise direction in Figure F-1). This full-circle progression projects the largest interval space—*ic* 0—within the clock.

If one *pc* is moved to any other, the remaining six *ics* 1-6 will be derived, those that have relatively smaller spaces than those of *ic* 0. As a result, interval class 0 becomes the largest space among all of the seven interval classes (0-6). Ordering the *ics* 0-6 with respect to their corresponding spaces from the smallest to the largest in Figure F-1 will be *ics* 1, 2, 3, 4, 5, 6, and then 0.

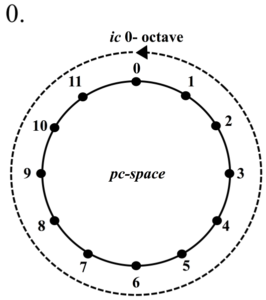


Figure F-1. The clock represents *pc-space.*

21. Later in the discussion of my theory of harmonic measurement, these seven *ics* {1, 2, 3, 4, 5, 6, 0} in Table 6 will be equivalent to voice-pair interval classes (*VPics*) {1, 2, 3, 4, 5, 6, 0}, respectively.

To summarize these two strengths, Table 5 creates a clear image of Seeger's primary (qualities: consonant/dissonant) and secondary (conditions: perfect/imperfect) divisions of the *ics*, and Table 6 further examines the spaces (compact/spatial) among these seven elements. In other words, two generators—quality and space—systematically and respectively arrange the *ics* in these two tables. Between these two generators, the latter is particularly significant: it distinguishes an *ic* in relation to its relative degree of compactness. The identity of *ic* 6 is no longer ambiguous since it always represents the second to the most spatial interval class.[22] Thus, if the focus of an *ic* is shifted from *quality* to *space*, the uncertain quality created by this particular interval class is avoided, and even solved. From here onwards, the interval classes will be distinguished solely by their relative degrees of compactness.

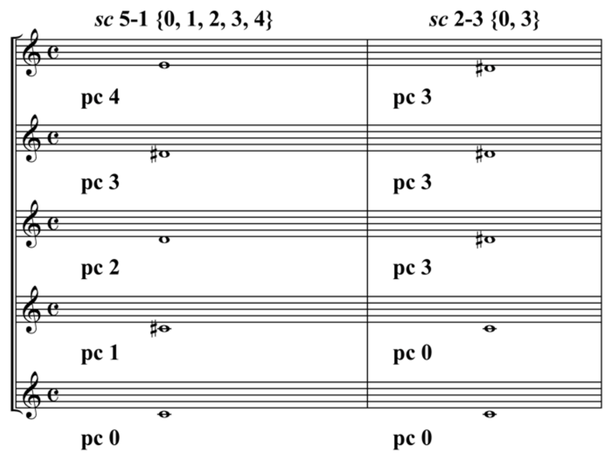
Earlier, I defined a harmony as a *VPicset* (voice-pair interval-class set) that contains several *VPics*. The proportion of the consonant *VPics* to the dissonant ones affects the quality of that harmony. However, based on the above conclusion, an *ic* (specifically, a *VPic*) is no longer distinguished by its quality, but by its relative compactness. Therefore, to support my observation in Table 6, I suggest that the proportion of the spatial *VPics* to the compact ones determines the degree of compactness of a harmony. This means that a harmony, whose *VPics* mostly appear on the extreme right of Table 6, is more spatial. In contrast, a more compact harmony is mainly composed of *VPics* on the extreme left of Table 6.

To examine the proportion between compact and spatial *VPics* within a *VPicset*, I propose a harmonic measurement: the "average voice-pair interval-class set" (*average-VPicset*). This measurement sums up all the *VPics* in one *VPicset* and then averages the total. The result is

22. In contrast, Table 5 distinguishes neither the quality nor the condition of *ic* 6. Although this interval class appears between the *ics* 5 and 0, it does not belong to the category of perfect consonance. Instead, it is labeled with a question mark to show the ambiguity of this special interval. To avoid and solve the ambiguity created by *ic* 6, attention must be directed to the other generator of this particular interval class: space.

an averaged number that represents the particular degree of compactness of a harmony: the larger the number, the more spatial the harmony, and the reverse: the smaller the number, the more compact the harmony. There is, however, a crucial problem in this harmonic measurement—the *average-VPicset* works *if and only if there is no VPic 0*.

For instance, in Example 11, if the *pcs* {0, 3} in the first chord of *sc* 5-1 are repeated and the remaining *pcs* {1, 2, 4} are moved by one semitone, the second chord of *sc* 2-3 is derived, which contains only two different *pcs* {0, 3} and creates an interval of a minor third. Examining these two *VPicsets* in Example 11, all four *VPics* in *sc* 2-3, {0, 3, 0, 0}, are more spatial than those in *sc* 5-1, {1, 1, 1, 1}, since both *VPics* 0 and 3 appear to the right of *VPic* 1 in Table 6. Summing up the corresponding *VPics* and averaging the total derives the *average-VPicsets* 1 (*sc* 5-1) and 0.75 (*sc* 2-3). Based on the above notion of "the smaller the *average-VPicset*, the more compact the harmony," this result suggests that *sc* 2-3 with the number 0.75 is more compact than *sc* 5-1, whose *average-VPicset* is 1. However, the image of *sc* 2-3 being more compact than *sc* 5-1 disagrees with my earlier observation, which reveals that all the *VPics* in *sc* 2-3 are more spatial than those in *sc* 5-1. The problem creating this conflict between my observation and the resultant *average-VPicsets* originates from the *exact value of the number*. Although the number "0" represents the most spatial *ic* in Table 6, its exact value is actually smaller than all the remaining "1" to "6." Thus, in the process of averaging the sum of *VPics*, zero will decrease the value of an *average-VPicset*. Using "0" to represent the most spatial interval class turns out to be irrelevant for my harmonic measurement, and the final result cannot satisfy the notion of "the smaller the *average-VPicset*, the more compact the harmony."



[*average-VPicset*] $\frac{1+1+1+1}{4} = 1$ $\frac{0+3+0+0}{4} = 0.75$

Example 11. *Average-VPicsets* of *scs* 5-1 and 2-3.

To solve the problem illustrated in Example 11, the *ics* from 1 to 0 in Table 6 are re-valued by subtracting one degree from each interval class. As a result, *ic* 1 is equal to 0, *ic* 2 is equal to 1, and so on, until *ic* 0 is equal to 6.[23] Observing these new numerical representations, the most compact *ic* 1 has the smallest value of 0, while the largest number 6 describes the most spatial *ic* 0. These revalued *ics* are "transformed interval classes" (*tics*), shown in Table 7.[24]

23. Since the seven sequential interval classes are recursive and form a closed group, the revalued numbers must also be sequential and recursive. Subtracting one degree from the first interval class (*ic* 1) in Table 6 derives the first number of 0. If the seven revalued sequential numbers begin with 0, the remaining six must be {1, 2, 3, 4, 5, 6}. As a result, the last interval class (*ic* 0) in Table 6 corresponds to the number of 6 that represents the most spatial *ic* among all seven interval classes.

24. Deriving the results in Table 7, the numbers in the top two rows progress "linearly," not "circularly." If these two orderings are perceived as loops, the numbers appearing on the extremes of this table would be connected, creating circles of *ics* (*VPics*) and *tics* (*TVPics*). This would mean the *ics* 0 (the most spatial) and 1 (the most compact) as well as *tics* 6 (the largest space) and 0 (the smallest space) are adjacent to each other, therefore weakening a clear boundary between the smallest and largest spaces. In fact, the numbers in the top two rows of Table 7 actually progress back and forth from one extreme to another. In this case, the two extremes can never meet each other, maintaining a firm boundary between the smallest and the largest *tics* (0 and 6), and between the most compact and the most spatial *ics* (1 and 0).

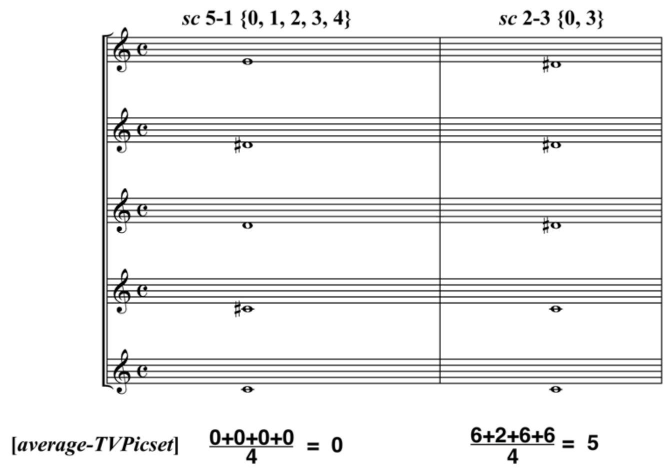
ics (VPics)	1	2	3	4	5	6	0
tics (TVPics)	0	1	2	3	4	5	6
The degree of compactness	compact → spatial						

Table 7. Translating the *ics* (*VPics*) in Table 4 into *tics* (*TVPics*).

The greatest strength of these new revalued numbers (*tics* 0–6) is their ability to provide a truer picture of "the smaller the *tic*, the more compact the *ic*." The "transformed voice-pair interval-class" (*TVPic*) is a *VPic* that is replaced by its corresponding *tic*, and several *TVPics* form a set (*TVPicset*). Similar to the *average-VPicset*, the average of a *TVPicset* will be a number—the average voice-pair interval-class set (*average- TVPicset*)—that accommodates the harmonic degree of compactness. This number, or *average-TVPicset*, consistently ranges from 0.00 to 6.00.[25] Within this range, the smaller the *average-TVPicset*, the more compact the harmony; the larger the *average-TVPicset*, the more spatial the harmony. If *TVPics* are used to replace *VPics* in Example 11, the *average-TVPicsets* are 0 (*sc* 5-1) and 5 (*sc* 2-3), shown in Example 12. Compared to the earlier result (*average-VPicsets* 1 and 0.75 in Example 11), these two new numbers (*average-TVPicsets* 0 and 5 in Example 12) interpret in a better way, the degrees of compactness of *scs* 5-1 and 2-3: five adjacent chromatic pitches are more compact than a minor third. To accurately compare the harmonic degrees of compactness, all *VPics* must be translated into *TVPics* before any further calculation. Since the *average-TVPicset* is a more

25. If a *TVPicset* is only composed of the most compact *TVPic* of 0, summing up all the *TVPics* and averaging the total derives the *average-TVPicset* of 0.00, which corresponds to the smallest number and represents the most compact harmony. On the contrary, if a *TVPicset* is only composed of the most spatial *TVPic* of 6, summing up all the *TVPics* and averaging the total derives the *average-TVPicset* of 6.00, which corresponds to the largest number and represents the most spatial harmony. Since the smallest and the largest *average-TVPicset*, respectively, are 0.00 and 6.00, connecting these two numbers with each other forms a limited range that covers all the possible *average-TPVicsets* derived from the application of my harmonic measurement.

reliable measurement than the *average-VPicset*, this newly derived measurement (*average-TVPicset*) is applied to examine the harmonic degrees of compactness.[26]

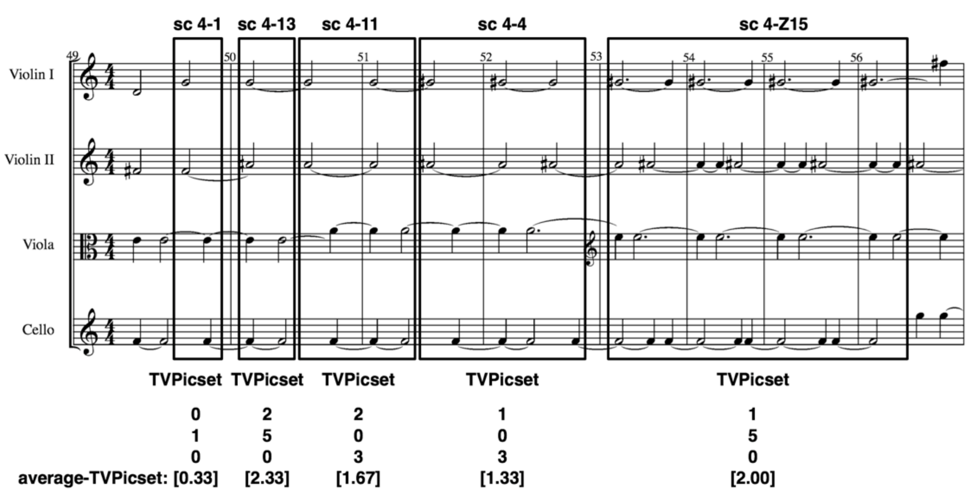


Example 12. *Average-TVPicsets* of *scs* 5-1 and 2-3.

Finally, the *average-TVPicset* is applied to examine the harmonic degrees of compactness of tetrachords in Examples 7 and 8, and the derived results are shown in Examples 13 and 14, respectively. Below each musical system two items are indicated: the *TPVicset* and the *average-TVPicset*. The *TVPicset* contains three successively vertical numbers of *TVPics* and the bracketed numbers below the *TVPicsets* are the *average-TVPicsets* that range from 0.33 to 2.33.

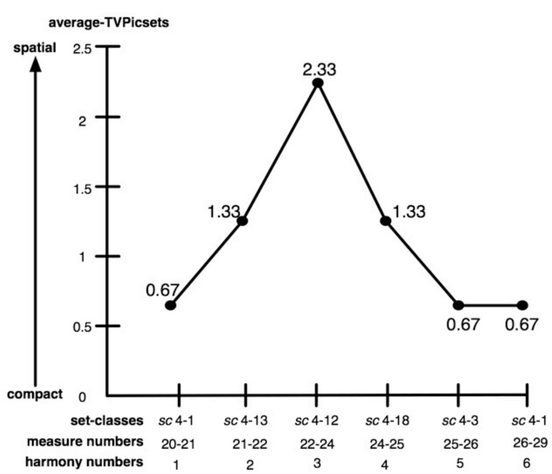
26. A similar approach to my harmonic measurement, but with a different theoretical concept, is found in Michael Klein's theory of the *compression of a harmonic aggregate*. Unlike my *average-TVPicset*, Klein uses pitch-space (*p-space*) instead of pitch-class space (*pc-space*). For detailed discussion of the *compression*, see Klein (1995) "A theoretical study of the late music of Witold Lutosławski" and (1999) "Texture, register, and their formal roles in the music of Witold Lutosławski."

Example 13. Ruth Crawford, *String Quartet*, Movement III, measures 20–29 (1931).
The *average-TVPicset* measures the degree of compactness of each tetrachord.

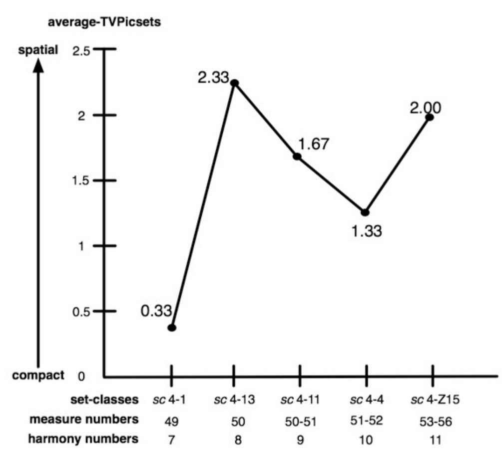


Example 14. Ruth Crawford, *String Quartet*, Movement III, measures 49–56 (1931).
The *average-TVPicset* measures the degree of compactness of each tetrachord.

To present the *average-TVPicsets* in Examples 13 and 14 in a way that mimics how the harmonies flow, I use a graph format (Graphs 1 and 2). The vertical axis in a graph marks the degrees of the *average-TVPicsets* from 0.00 to 2.50, from the most compact to the most spatial harmonies. The *sc* names and the measure numbers, in turn, appear on the horizontal axis. Below the measure numbers on the horizontal axis appears one more item: the harmony number (*HN*) that identifies its associated chord. Harmony numbers 1–6 appear in Graph 1 and *HNs* 7–11 appear in Graph 2. The dots are the *average-TVPicsets* of their corresponding tetrachords. Connecting the successive dots in one of the graphs forms a figure that represents the harmonic progression for its corresponding example.

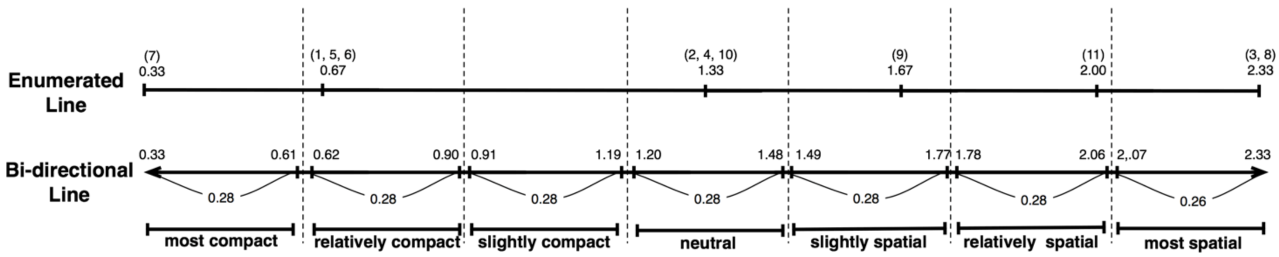


Graph 1. Graphical representation of the *average-TVPicsets* for Example 13; *HNs* 1–6.



Graph 2. Graphical representation of the *average-TVPicsets* for Example 14; *HNs* 7–11.

To accurately describe the relative degrees of compactness amongst the harmonies in Graphs 1 and 2, a way of determining them to be either spatial or compact in relation to their corresponding *average-TVPicsets* must be defined. Example 15 uses an enumerated line to represent all the *average-TVPicsets* in Graphs 1 and 2. The numbers in parentheses above each *average-TVPicset* represent their corresponding harmony numbers. This enumerated line is framed by the most compact (the *average-TVPicset* of 0.33) and the most spatial harmony (the *average-TVPicset* of 2.33). A bi-directional line spans the same distance as the enumerated line from 0.33 to 2.33.



Example 15. Spatial and compact harmonies are defined in relation to their *average-TVPicsets* in Graphs 1 and 2; numbers in parentheses are the *HNs*.

This bi-directional line is divided equally into seven sections, and each one spans 0.28 degrees.[27] From left to right, the seven sections successively range from 0.33–0.61, 0.62–0.90, 0.91–1.19, 1.20–1.48, 1.49–1.77, 1.78–2.06, and 2.07–2.33. Based on the notion of "the smaller the *average-TVPicset*, the more compact the harmony," each section is defined according to its range as describing a particularly harmonic degree of compactness: the most compact 0.33–0.61, the relatively compact 0.62–0.90, the slightly compact 0.91–1.19, the neutral 1.20– 1.48, the slightly spatial 1.49–1.77, the relatively spatial 1.78–2.06, and the most spatial 2.07– 2.33.

Using these seven sections in Example 15 as the measuring references, the divisions are extended in the bi-directional line to meet the enumerated line by adding six vertical dotted lines. With these additional marks, the enumerated line is divided into seven sections, identical to those in the bi-directional line. Instead of the slightly compact section, each section contains one

27. The only exceptional section is the one on the extreme right that spans 0.26 degrees.

average-TVPicset that demonstrates a particularly harmonic degree of compactness. Thus, from left to right on the enumerated line, these seven successive sections along with their corresponding *average-TVPicsets* are the most compact 0.33 (*HN* 7), the relatively compact 0.67 (*HNs* 1, 5, 6), the neutral 1.33 (*HNs* 2, 4, 10), the slightly spatial 1.67 (*HN* 9), the relatively spatial 2.00 (*HN* 11), and the most spatial 2.00 (*HNs* 3, 8). Grounded in the framework of this enumerated line, a specific degree of compactness is assigned to each harmony in the following analysis.

Graph 1 contains six dots (*HNs* 1–6) that are from three different sections on the enumerated line in Example 15: the relatively compact (*HNs* 1, 5, and 6), the neutral (*HNs* 2 and 4), and the most spatial (*HN* 3). Connecting all the six successive dots creates an arch that represents the harmonic progression for Example 13. The first half of the arch forms an ascending line that begins with the relatively compact chord (*HN* 1) and gradually rises to the most spatial chord (*HN* 3) by passing through a neutral chord, *HN 2*. However, reaching the higher range of the graph is only temporary, because immediately following the ascent to *HN* 3, the arch inverts this line and creates a symmetrical descent that begins with the most spatial chord (*HN* 3) and gradually descends to the relatively compact chord (*HN* 5) by passing through a neutral chord (*HN* 4). After *HN* 5, the arch maintains its motion by progressing to one more chord (*HN* 6), whose harmonic degree of compactness is the same as that of *HN* 5, the relatively compact chord. Based on the analysis of Graph 1, the arch is now defined as the harmonic prolongation of the relatively compact chord because it begins and ends with the relatively compact chords (*HNs* 1, 5, 6) and reserves the neutral (*HNs* 2, 4) and the most spatial ones (*HN* 3) for the middle.

There are two essential features in Graph 2 that distinguish it from Graph 1. First, each of the five dots in Graph 2 represents a chord with a specific and different harmonic degree of compactness. From the smallest to the largest *average-TVPicsets*, these dots correspond to the most compact (*HN* 7), the neutral (*HN* 10), the slightly spatial (*HN* 9), the relatively spatial (*HN* 11), and the most spatial chords (*HN* 8). Second, these five successive dots connected, form a wave; a figure, different from the arch in Graph 1, that projects the harmonic progression for Example 14. It begins with a direct ascent from a dot in the lowest range of the graph (*HN* 7, the most compact) to that in the highest range (*HN* 8, the most spatial). As soon as the wave arrives at *HN* 8, the remainder of its progression becomes comparatively smoother, as it moderately descends to a neutral chord (*HN* 10) by passing through a slightly spatial chord (*HN* 9) and then rises again to a dot at the higher range of the graph at *HN* 11, the relatively spatial chord. This wave in Graph 2 is an interesting harmonic progression opening with the most compact and the most spatial chords (*HN*s 7–8) and leaving behind those with rather mediate degrees of compactness (i.e., from the neutral to the relatively spatial chords).

Through these analyses of Graphs 1 and 2, it is clear that each graph contains a particular figure that projects a unique harmonic progression for its associated musical passage. In the first passage (Graph 1 and Example 13) Crawford distributes the relatively compact (*HNs* 1, 5, and 6) and the most spatial chords (*HN* 3) over the entire harmonic progression, and forms a gradual ascent and descent that covers the other two neutral chords (*HNs* 2 and 4). Contrarily, in the second passage (Graph 2 and Example 14) Crawford uses a different harmonic progression, where the most compact and spatial chords are adjacent to each other (*HNs* 7 and 8) and forms a considerable ascent at the very beginning of the passage. Unlike the arch in Graph 1, this ascent is not deviated or interrupted by any other chords with rather mediate degrees of compactness, because they (*HNs* 9–11) all appear after the initial ascent in Graph 2. Based on the observations and comparisons between Graphs 1 and 2, I lead these findings and discussion into conclusion.

CONCLUSION

In the third movement from Crawford's dissonant contrapuntal composition, *String Quartet*, there are two musical passages that share several common contextual features. These features include the initial *scs* 4-1 and 4-13, a meter of 4/4, and the long duration of the final tetrachord. On the surface, these common contextual features suggest that both passages unfold a similar or related musical progression. However, beneath these features, a grounded element—a harmonic progression—is revealed that can better describe an underlying relationship between these two passages. While my analyses, based on Seeger's pre-compositional rules and classification of intervals, cannot exactly determine whether the harmonic progressions of these two passages are similar to each other, the application of the *average-TVPicset* (which takes into account the spaces of all intervals) to re-analyze the harmonies within these two passages derives one decided and appealing result: two markedly different harmonic progressions. They point out a significant dissimilarity that distinguishes one passage from the other. Although Crawford uses several common surface features to project these two musical passages, she nonetheless carefully and deliberately varies their bases by superimposing them on two different harmonic progressions. Thus, I propose the term "coexistence of similarity (contextual features) and difference (harmonic progressions)" to describe my theoretical analyses of dissonant counterpoint as displayed in Crawford's *String Quartet*.

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